Causal Inference for Health Data

(STATS C160/C260 — Winter 2026)

Lecture 14:
Variation Analysis & Health Equity

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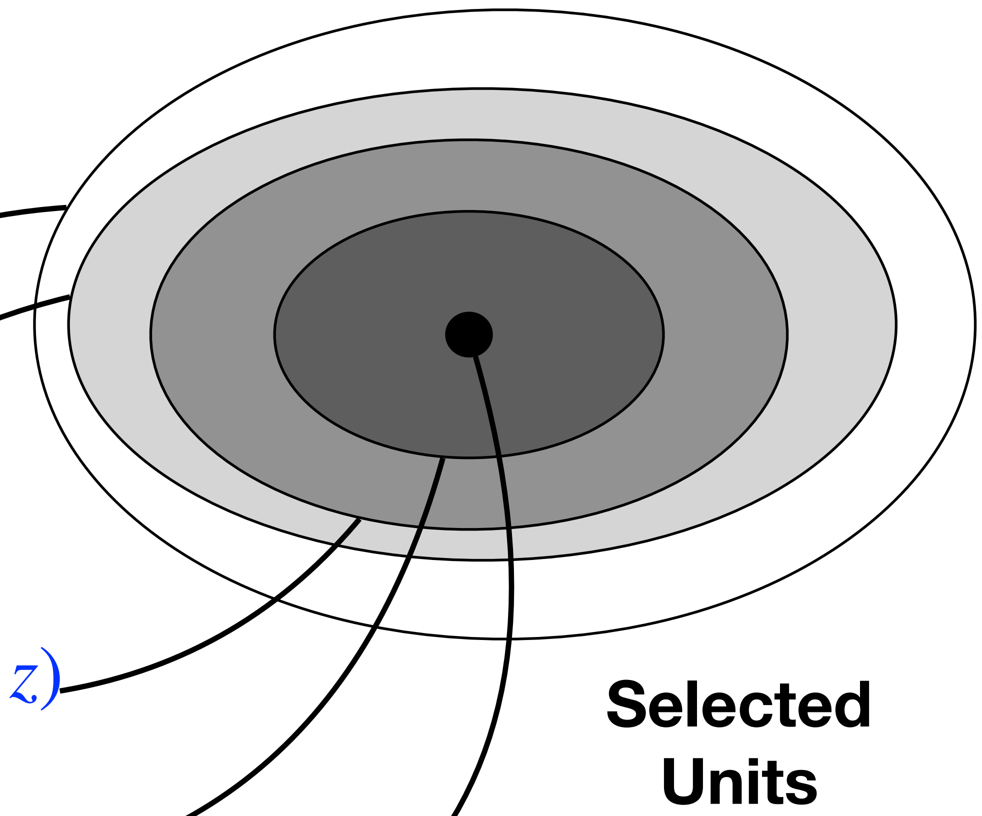
CAUSAL INFERENCE FOR HEALTH DATA

Recap: Granularity

Quantity	Unit-level Difference	Posterior
TE	$\mathbf{Y}_{X=x_1}(\mathbf{u}) - \mathbf{Y}_{X=x_0}(\mathbf{u})$	$P(\mathbf{u})$
ETT	$\mathbf{Y}_{X=x_1}(\mathbf{u}) - \mathbf{Y}_{X=x_0}(\mathbf{u})$	$P(\mathbf{u} \mid X = x)$
x, z-TE	$\mathbf{Y}_{X=x_1}(\mathbf{u}) - \mathbf{Y}_{X=x_0}(\mathbf{u})$	$P(\mathbf{u} \mid X = x, Z = z)$
v-TE	$\mathbf{Y}_{X=x_1}(\mathbf{u}) - \mathbf{Y}_{X=x_0}(\mathbf{u})$	$P(\mathbf{u} \mid V = v)$
u-TE	$\mathbf{Y}_{X=x_1}(\mathbf{u}) - \mathbf{Y}_{X=x_0}(\mathbf{u})$	$\delta_{\mathbf{u}}$



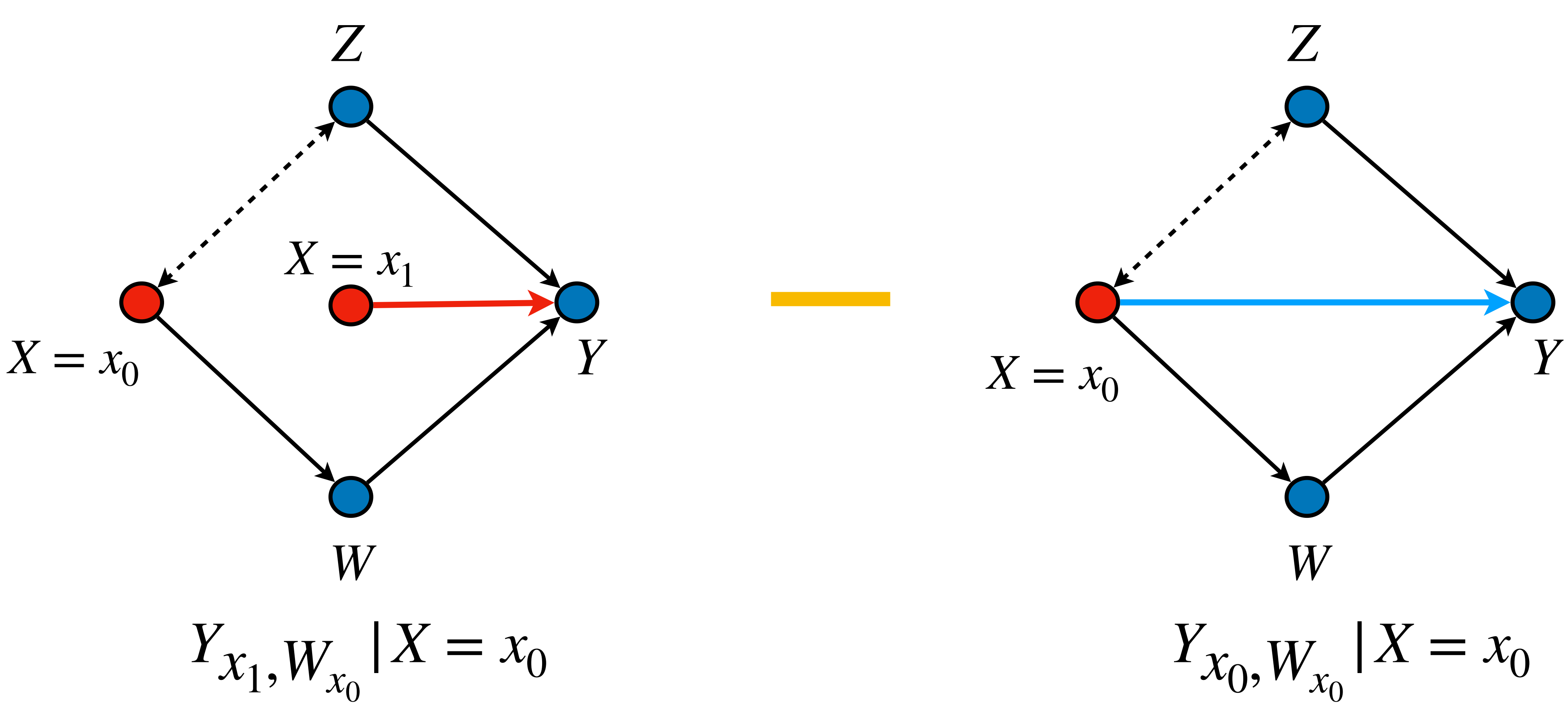
Generic Format:

$$\sum_{\mathbf{u}} [\mathbf{Y}_{X=x_1}(\mathbf{u}) - \mathbf{Y}_{X=x_0}(\mathbf{u})] \times P(\mathbf{u} \mid \mathbf{E} = \mathbf{e})$$

Gedankenexperiment (x-DE)

- For a patient *observed* without treatment ($X = x_0$), how would their HbA1c level (Y) change had they been given treatment ($X = x_1$), while keeping the weight loss unchanged (at the natural level $X = x_0$)?

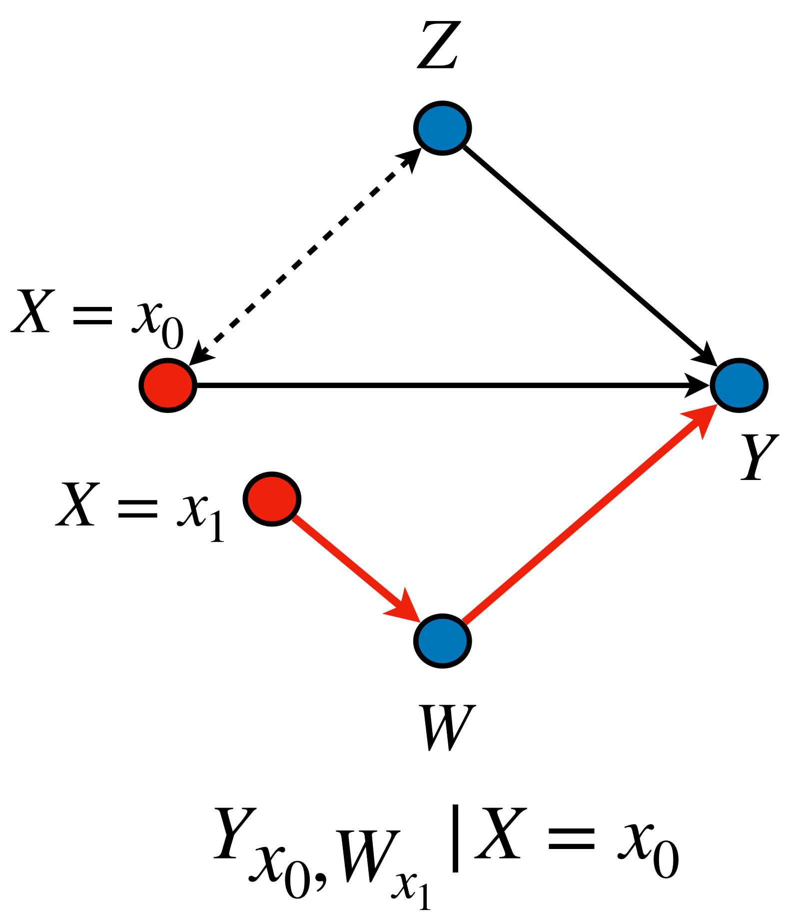
$$x\textbf{-DE}_{x_0,x_1}(y) = P(y_{x_1, W_{x_0}} \mid x_0) - P(y_{x_0, W_{x_0}} \mid x_0)$$



Gedankenexperiment (x-IE)

- For a patient *observed* not to be treated ($X = x_0$), how would their HbA1c level (Y) change had they been treated ($X = x_1$), while keeping treatment unchanged along the direct causal pathway (at the natural level $X = x_0$)?

$$x\text{-}\mathbf{IE}_{x_0,x_1}(y) = P(y_{x_0,W_{x_1}} \mid x_0) - P(y_{x_0,W_{x_0}} \mid x_0)$$



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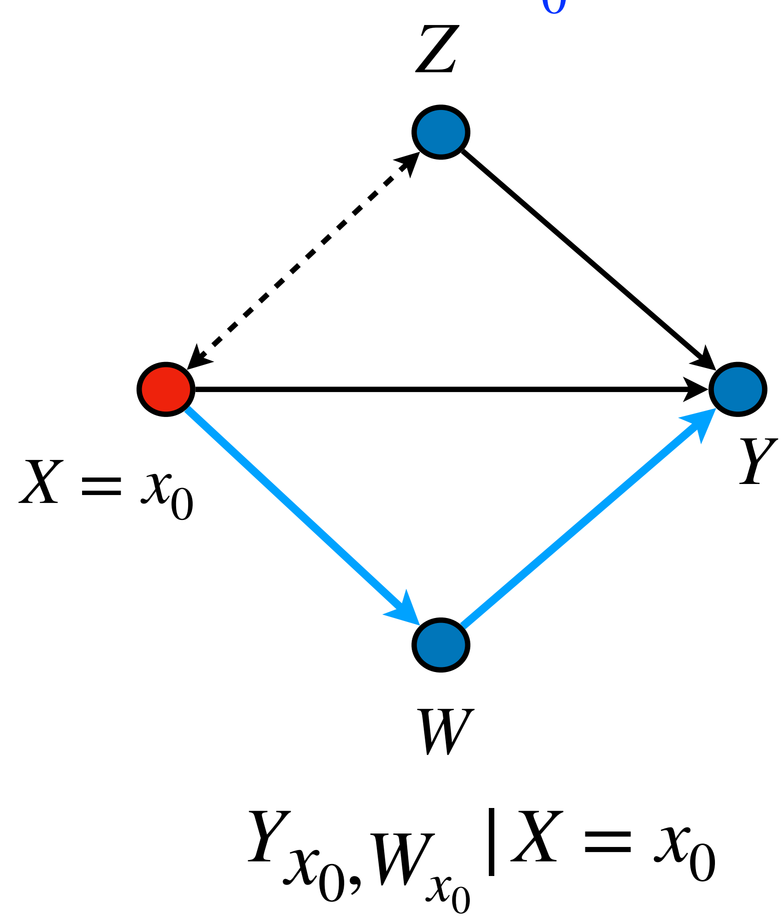
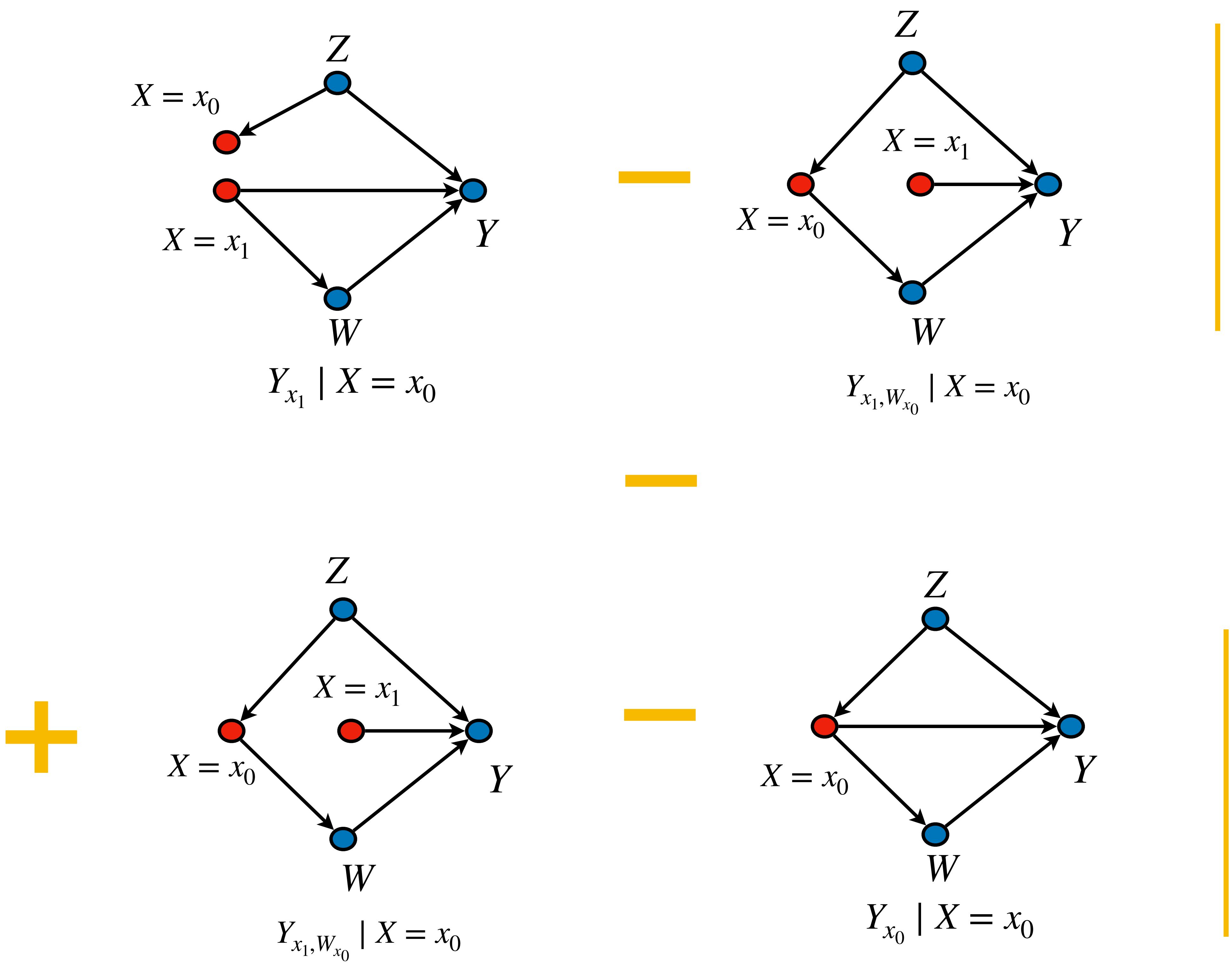


ETT Decomposition



$-x\text{-IE}_{x_1,x_0}(y \mid x_0)$

$\text{ETT}_{x_0,x_1}(Y \mid x_0)$

$x\text{-DE}_{x_0,x_1}(y \mid x_0)$

ETT Decomposition

$-x\text{-IE}_{x_1,x_0}(y \mid x_0)$

Theorem. *The effect of treatment on the treated can be decomposed into its direct and indirect parts:*

$$ETT_{x_0,x_1}(y \mid x_0) = x\text{-}DE_{x_0,x_1}(y \mid x_0) - x\text{-}IE_{x_1,x_0}(y \mid x_0)\,.$$

$\mid x_0)$

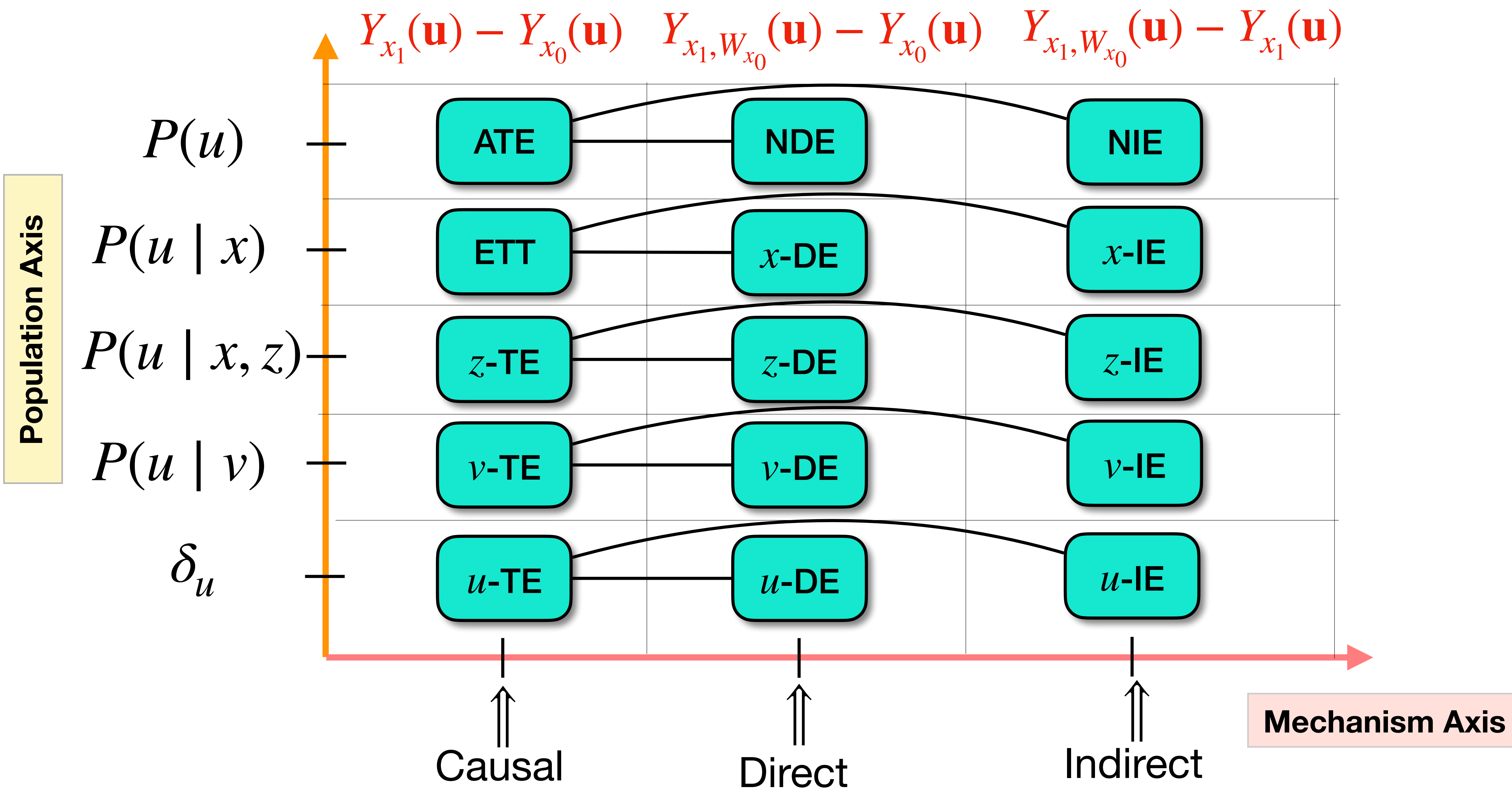
+

$Y_{x_1,W_{x_0}} \mid X = x_0$

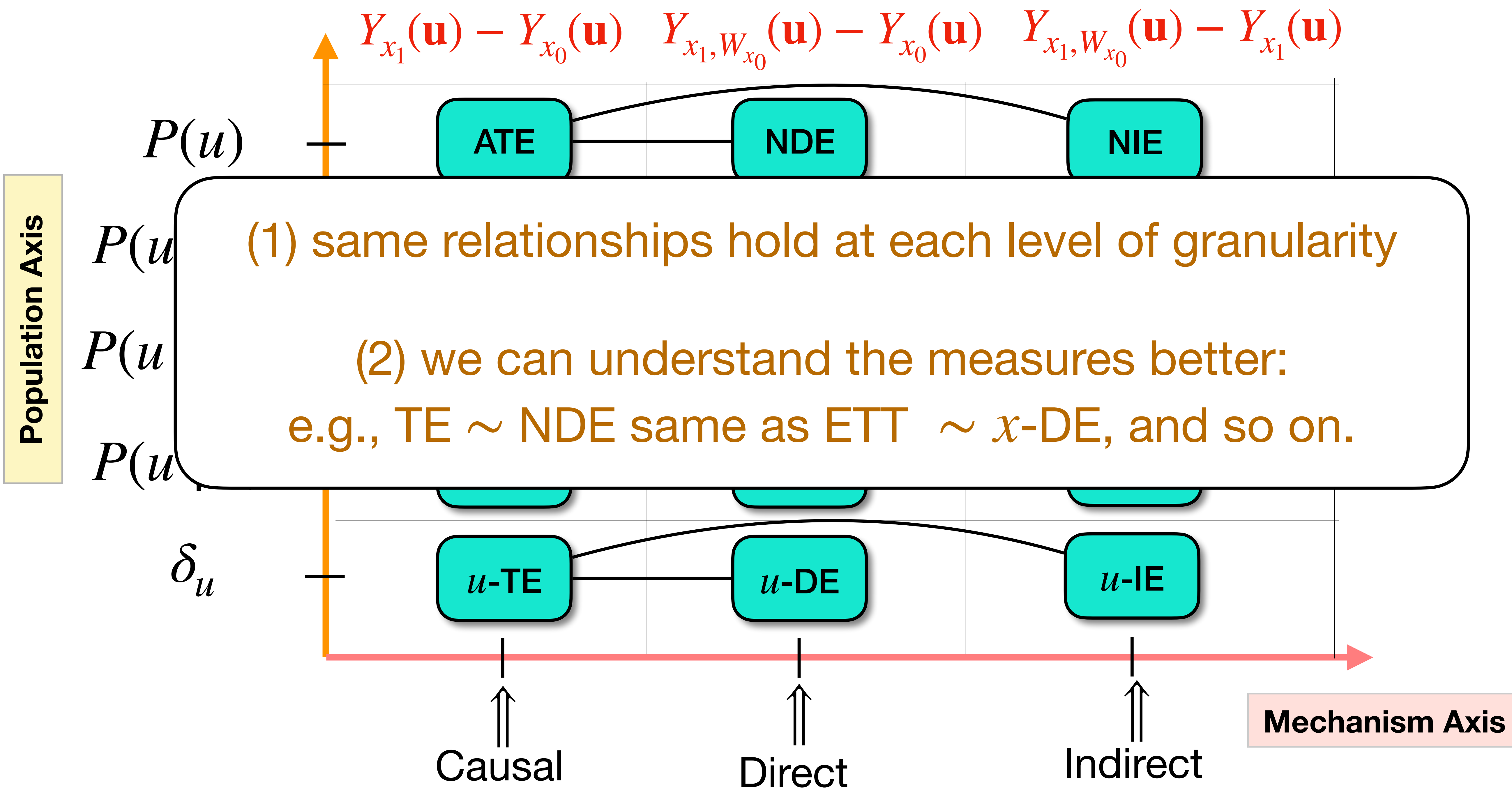
$Y_{x_0} \mid X = x_0$

$x\text{-DE}_{x_0,x_1}(y \mid x_0)$

Explainability Plane — v1



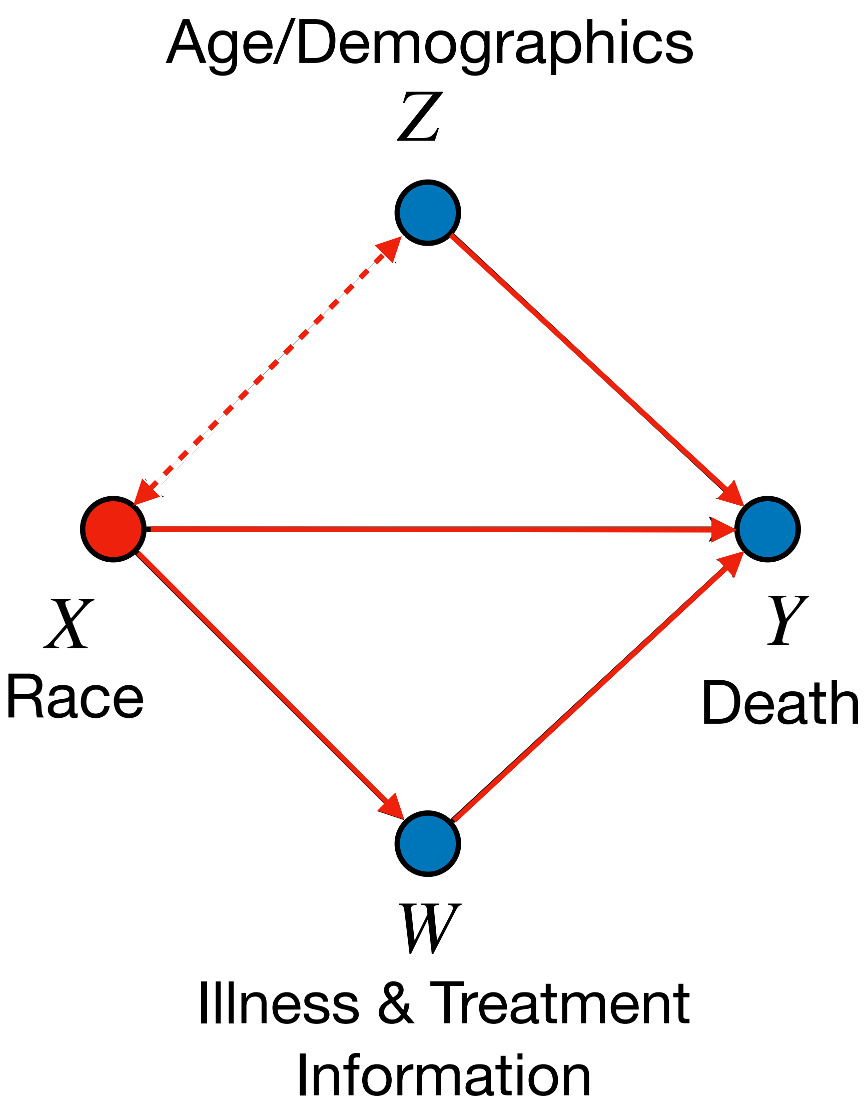
Explainability Plane — v1



(1) same relationships hold at each level of granularity

(2) we can understand the measures better:
e.g., TE ~ NDE same as ETT ~ x-DE, and so on.

Example: Mortality in ICU



- There is an observed disparity in mortality rates

$$\text{TV} = E[Y \mid x_1] - E[Y \mid x_0] = 0.8\,\%$$

between White (x_1) and African-American (x_0) patients. It could be explained in different ways, i.e.,

(1) The outcome is influenced directly by race: $X \rightarrow Y$.

(2) The outcome is a consequence of illness severity, which is affected by race: $X \rightarrow W \rightarrow Y$.

(3) Age or demographics, correlated with race, are affecting the outcome: $X \leftrightarrow Z \rightarrow Y$.

How to disentangle these variations within TV?

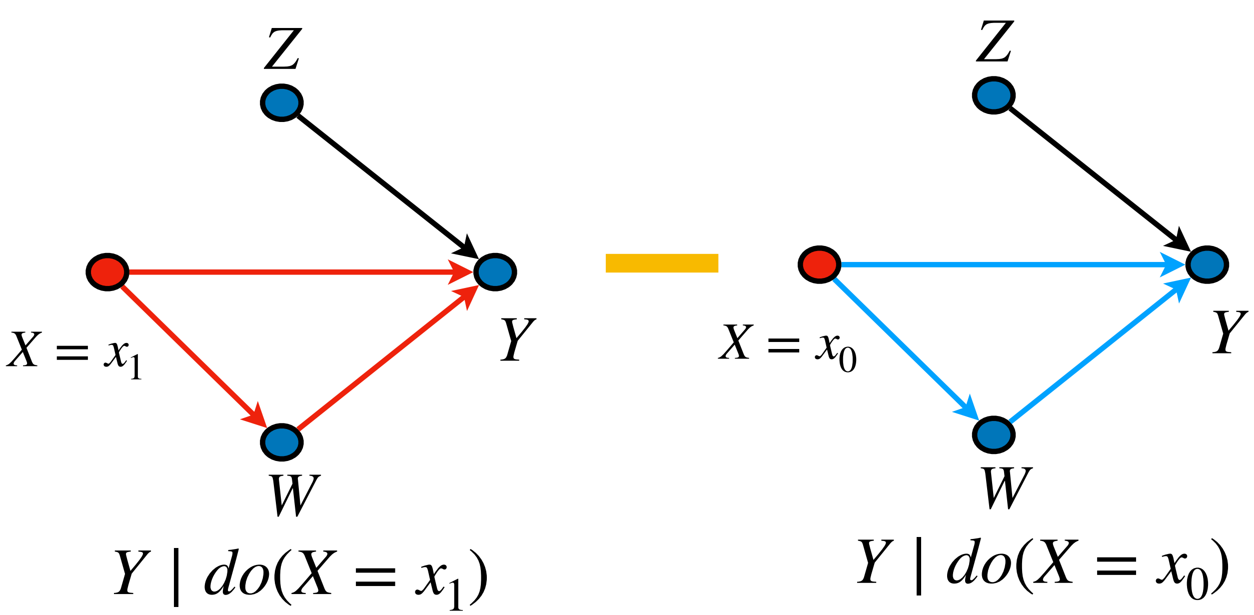
Total Variation

- The **Total Variation** $E(Y \mid X = x_1) - E(Y \mid X = x_0)$ is an L_1 -quantity representing the conditional difference of Y given $X = x_1$ vs. $X = x_0$.
- It does not invoke any interventions, but simply looks at units $\mathbf{U}$ s.t. $X = x_1, X = x_0$, naturally.
- The structural basis expansion of TV is:

$$TV_{x_0,x_1}(y) = \sum_{\mathbf{u}} \mathbf{Y}(\mathbf{u})[P(\mathbf{u} \mid X = x_1) - P(\mathbf{u} \mid X = x_0)]$$

TV vs. TE?

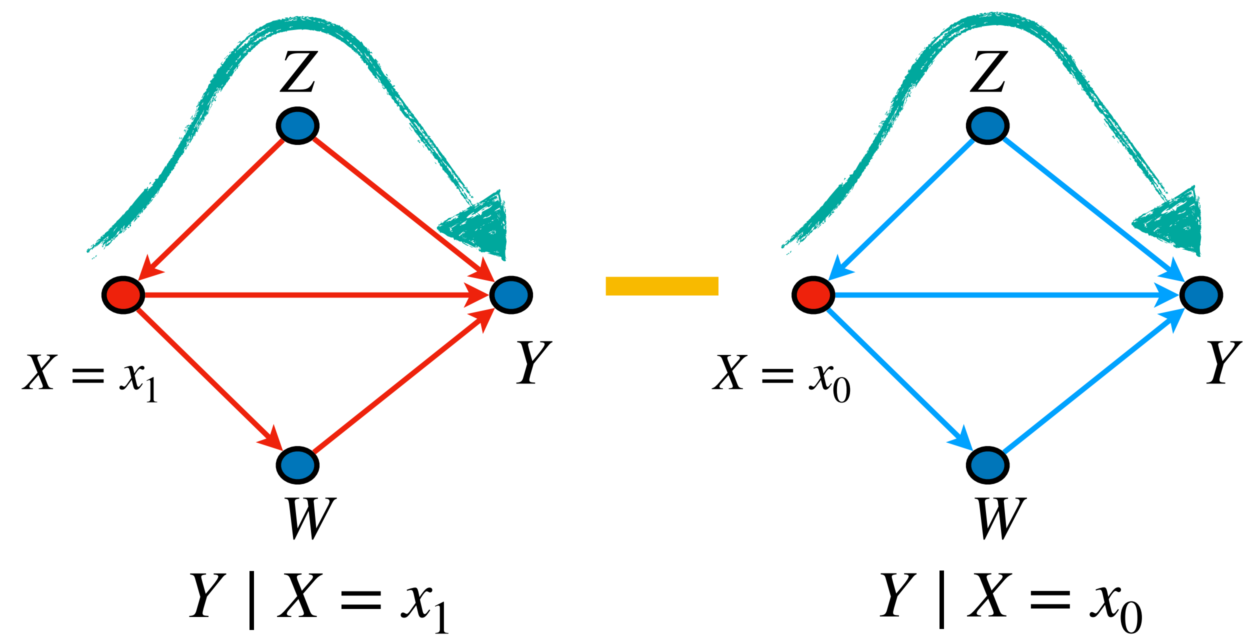
Total Effect



$$\textbf{TE}_{x_0,x_1}(y) = \sum_{\mathbf{u}} [Y_{x_1}(\mathbf{u}) - Y_{x_0}(\mathbf{u})]P(\mathbf{u})$$

Captures direct+indirect variations:
causal, downstream from X

Total Variation

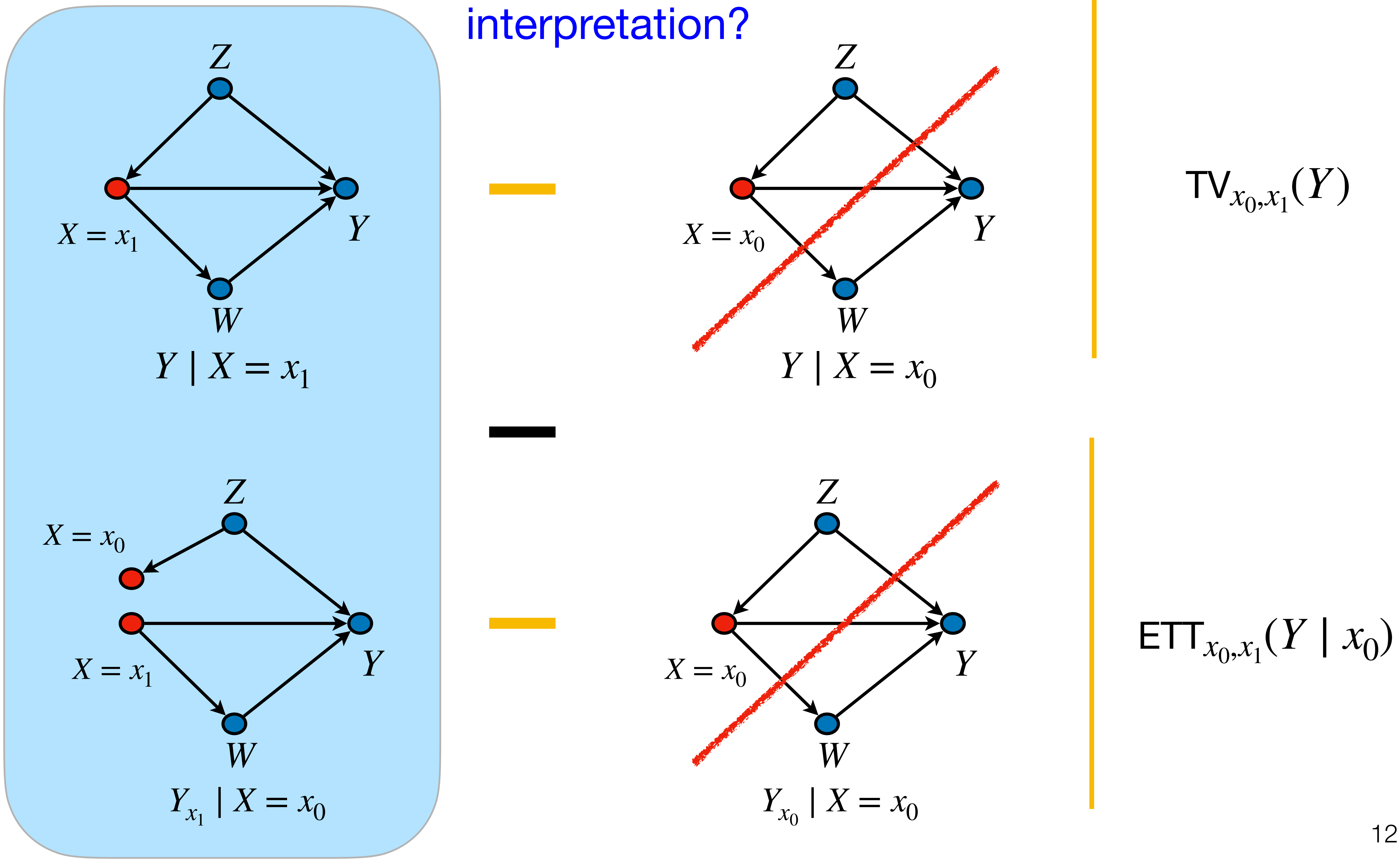


$$\textbf{TV}_{x_0,x_1}(y) = \sum_{\mathbf{u}} Y(\mathbf{u})[P(\mathbf{u} \mid x_1) - P(\mathbf{u} \mid x_0)]$$

Captures direct+indirect variations;
but also back-door (spurious)

Can we isolate spurious variations? TV — ETT

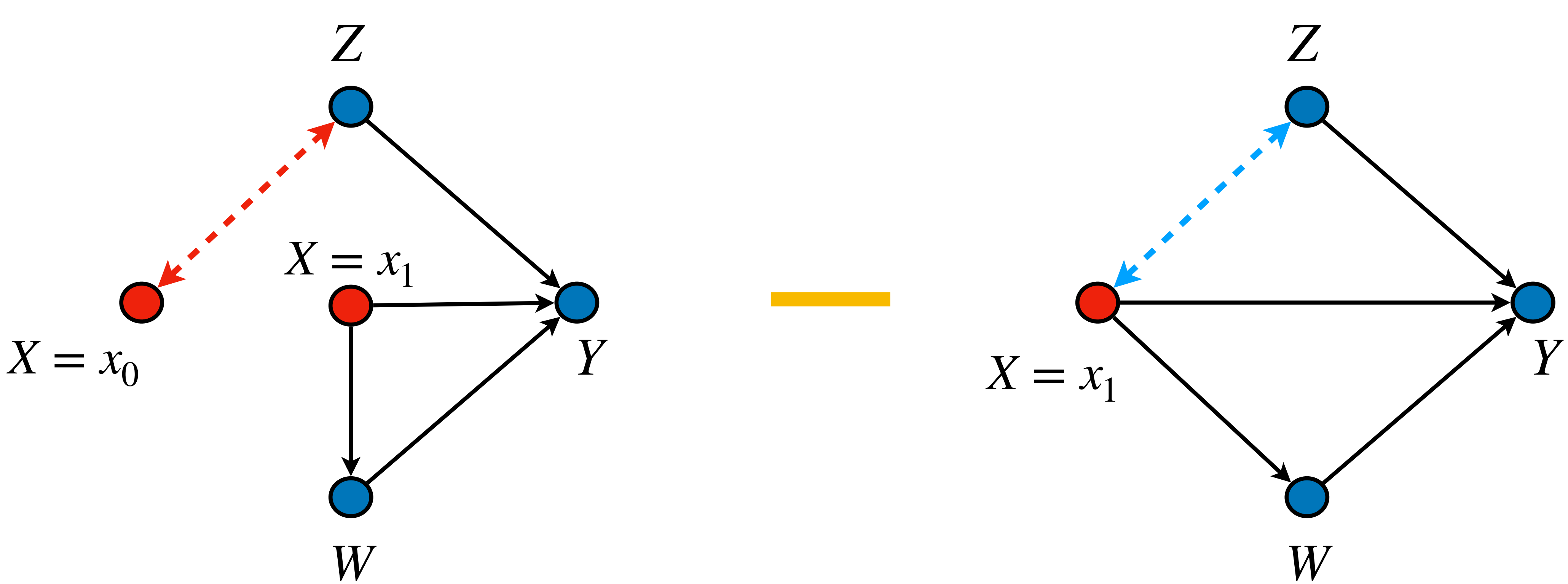
does this quantity have an interpretation?



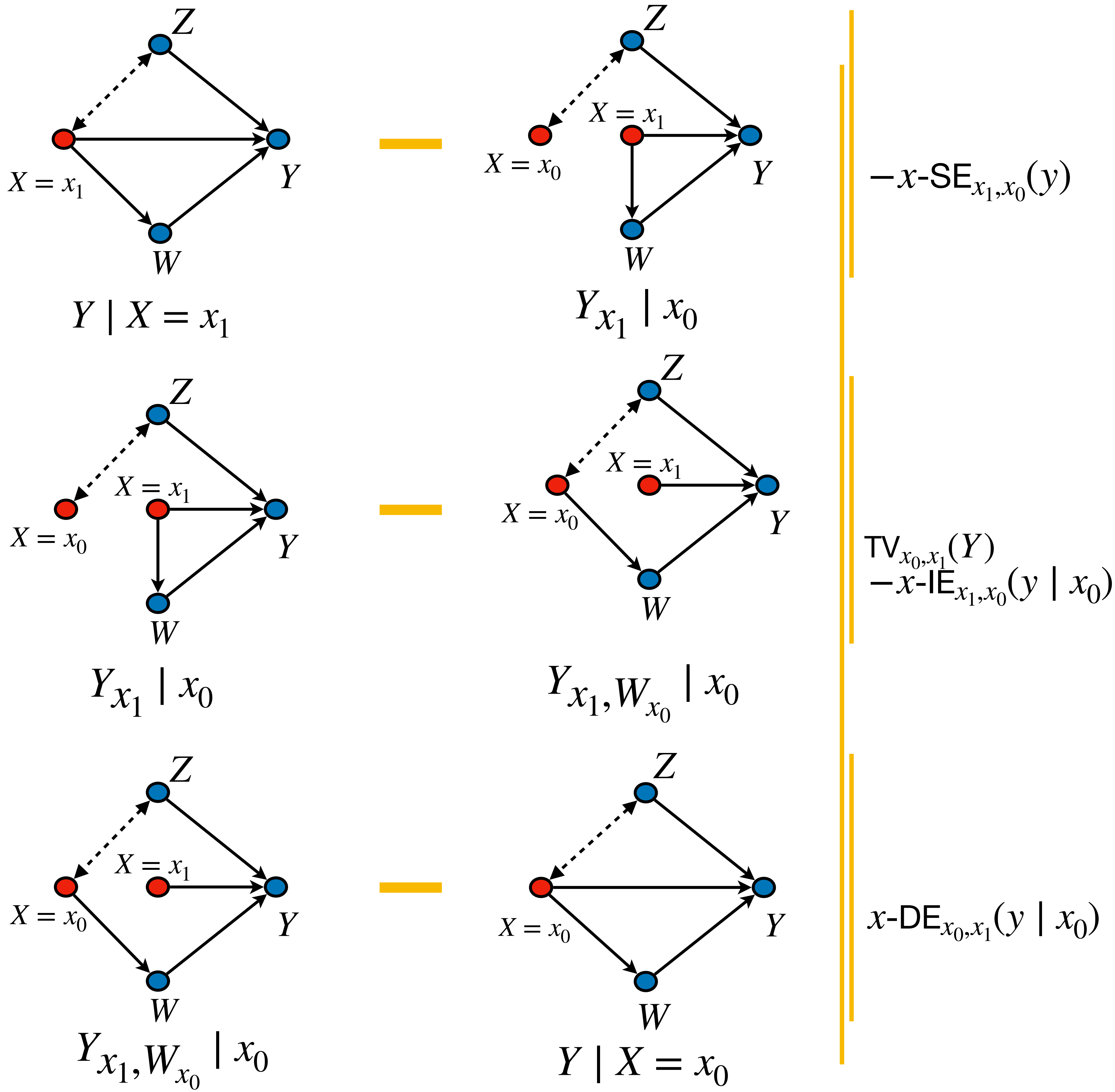
Gedankenexperiment (x-SE)

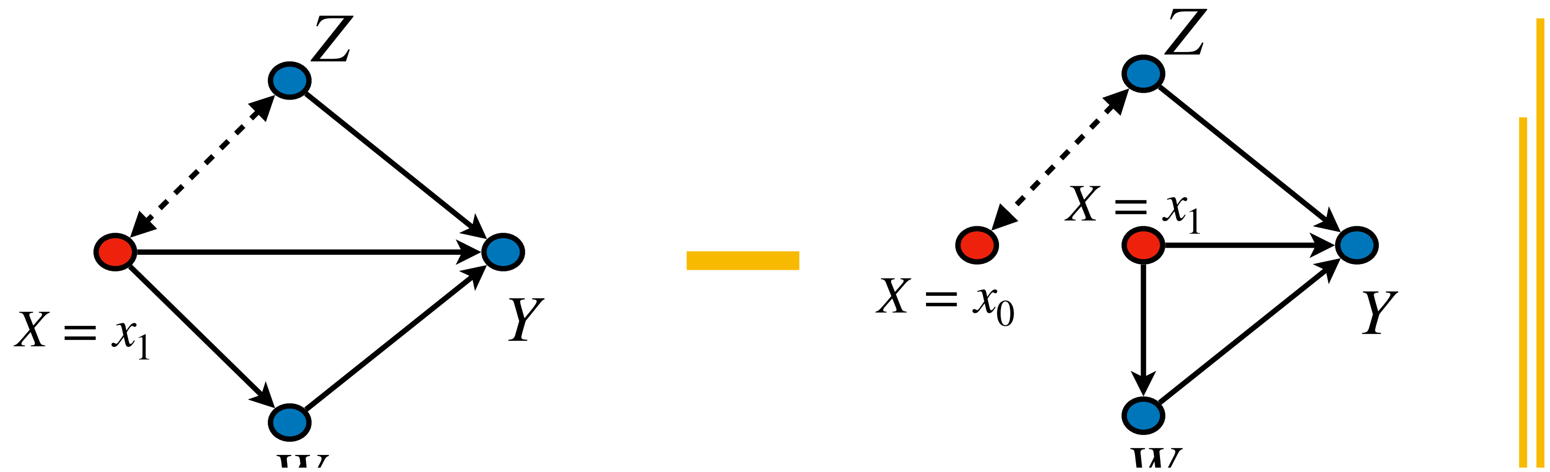
- For Majority group individuals ($X = x_1$) and Minority individuals ($X = x_0$), how would their mortality (Y) differ had they both been set to the Majority group by intervention ($X = x_1$)?

$$x\text{-}\mathbf{SE}_{x_1,x_0}(y) = P(y_{x_1} \mid x_0) - P(y_{x_1} \mid x_1)$$



$Y_{x_1} | X = x_0$ $\quad - \quad$ $Y_{x_1} | X = x_1$

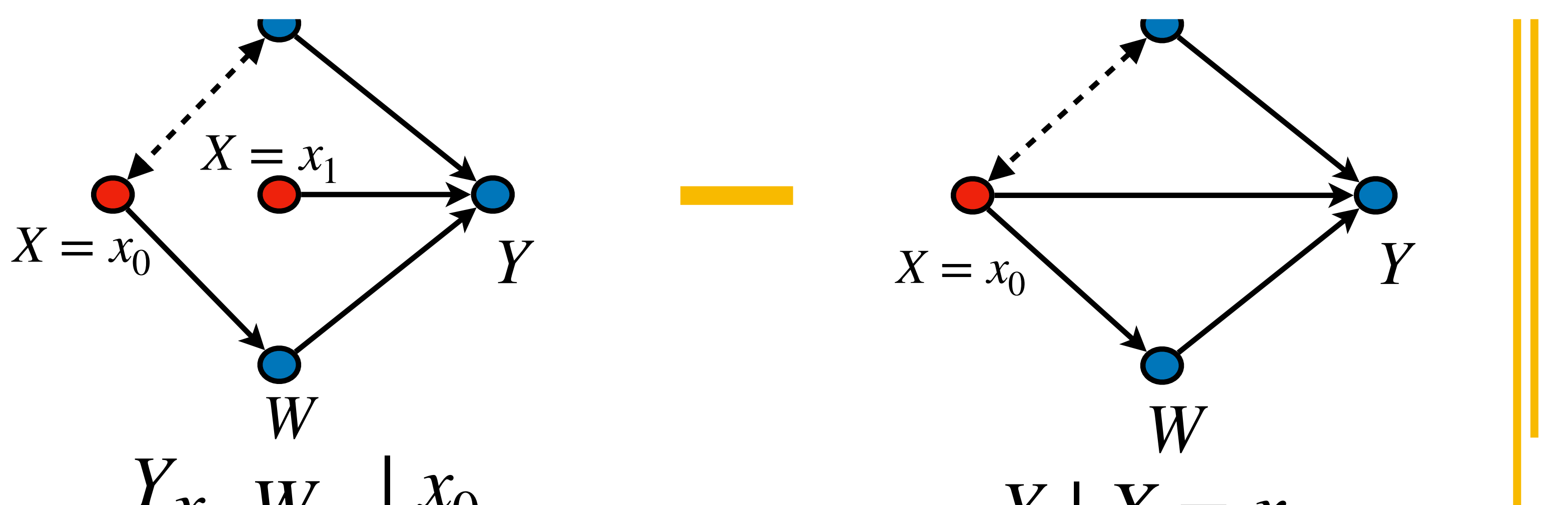

Z
X = x1
Y
W
Y | X = x1
X = x0
Yx1 | x0
Yx1,Wx0 | x0
Y | X = x0
−x-SEx1,x0(y)
TVx0,x1(Y)
−x-IEx1,x0(y | x0)
x-DEx0,x1(y | x0)



$-x\text{-SE}_{x_1,x_0}(y)$

Theorem. *The total variation measure can be decomposed into its direct, indirect, and spurious variations:*

$$TV_{x_0,x_1}(y) = \underbrace{x\text{-}DE_{x_0,x_1}(y \mid x_0)}_{direct} - \underbrace{x\text{-}IE_{x_1,x_0}(y \mid x_0)}_{indirect} - \underbrace{x\text{-}SE_{x_1,x_0}(y)}_{spurious}.$$



$Y_{x_1,W_{x_0}} \mid x_0$

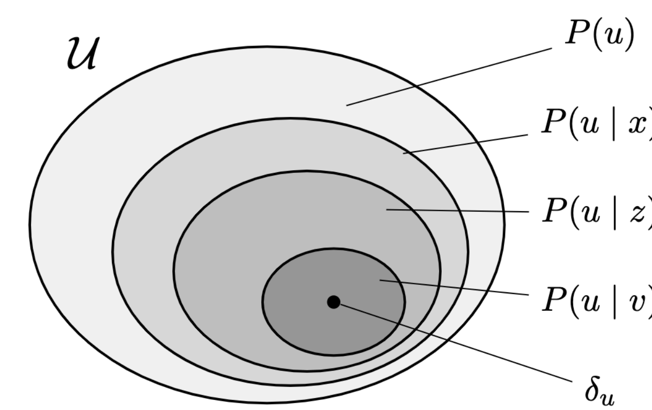
$Y \mid X = x_0$

$x\text{-DE}_{x_0,x_1}(y \mid x_0)$

Structural Basis Expansion: Causal Measures

Theorem (continued). *Whenever* $E_0 = E_1 = e$*, any counterfactual contrast* $P(y_{C_1} \mid E = e) - P(y_{C_0} \mid E = e)$ *admits the following structural basis expansion*

$$\sum_u \underbrace{[y_{C_1}(u) - y_{C_0}(u)]}_{\text{unit-level difference}} \; \underbrace{P(u \mid E = e)}_{\text{posterior}} .$$



For a specific unit $U = u$, Y's response to the transition $C_0 \rightarrow C_1$.

Population of units consistent with the factual evidence *E*=e.

Structural Basis Expansion: Spurious Measures

Theorem (continued). Whenever $C_0 = C_1 = c$*, any factual contrast* $P(y_c \mid E_1) - P(y_c \mid E_0)$ *admits the following structural basis expansion:*

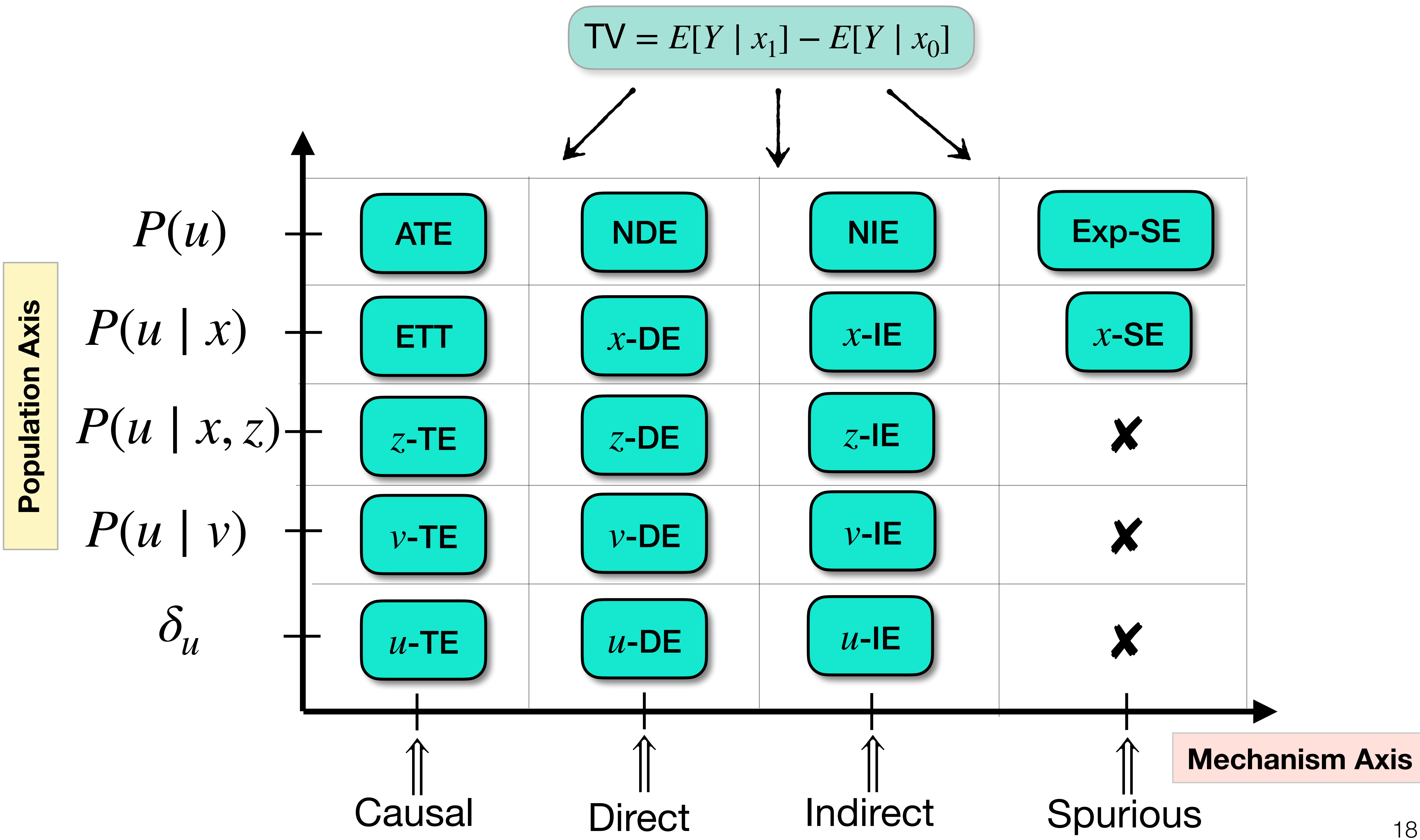
$$\sum_u \underbrace{y_c(u)}_{\text{unit outcome}} \underbrace{[P(u \mid E_1) - P(u \mid E_0)]}_{\text{posterior difference}}.$$

unit outcome → Baseline outcome for a fixed unit $U = u$.

posterior difference → Difference in posteria of how likely unit $U = u$ is selected under events E_0 vs. E_1.

- We will be mostly interested in contrasts w/ $C = x$, so that $X = x$ represents causal pathways.

Explainability Plane — v2



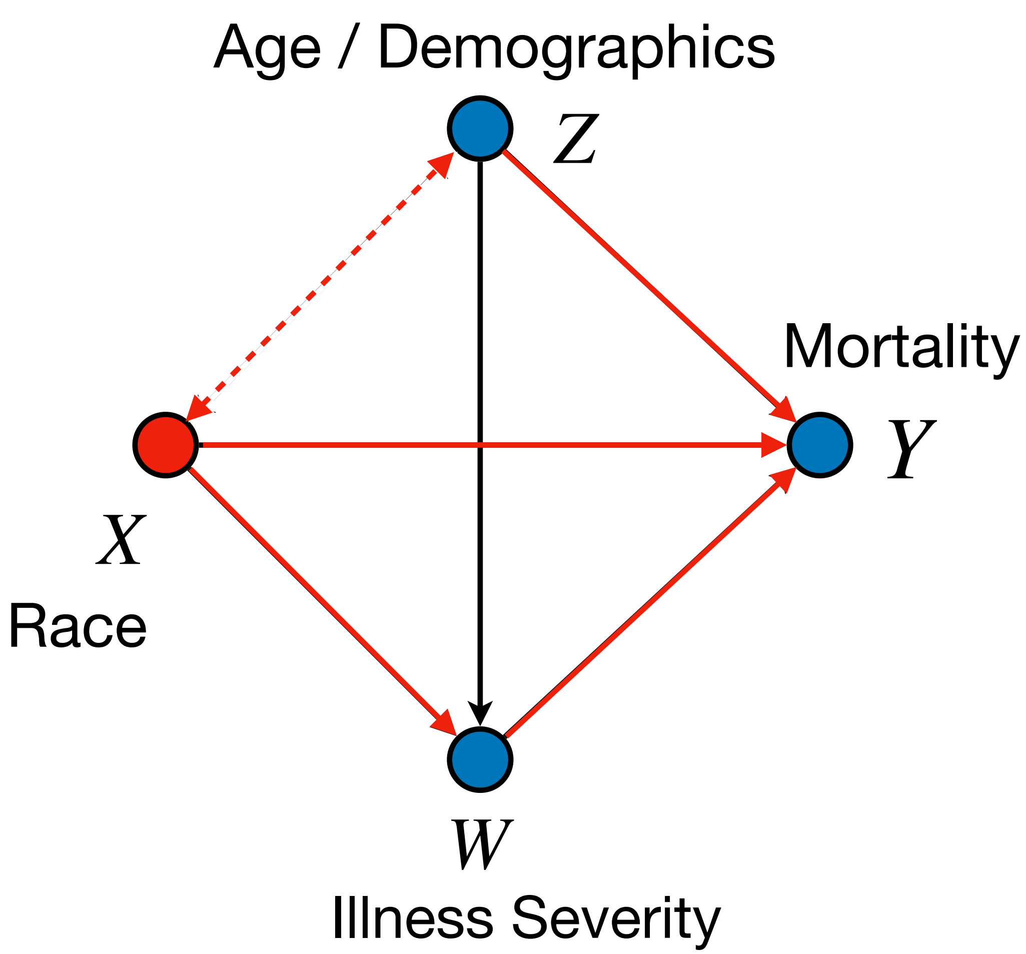
Bias Detection: Hospital Mortality after ICU

$E[Y \mid \text{White}] - E[Y \mid \text{African-American}] = 0.8\,\%.$

$E[Y \mid \text{Majority}] - E[Y \mid \text{Indigenous}] = -1.5\,\%.$

a — **Do similar mortality rates imply anything about equity?**

b — **Perform a causal explanation**

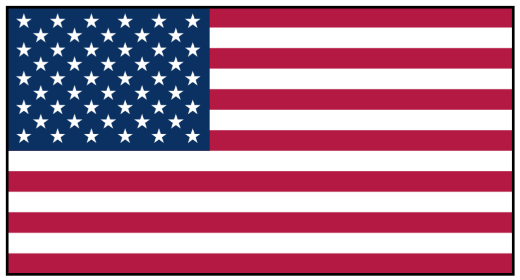


$$E[Y \mid x_1] - E[Y \mid x_0] = \underbrace{\textbf{DE}}_{X \to Y} + \underbrace{\textbf{IE}}_{X \to W \to Y} + \underbrace{\textbf{SE}}_{X \leftarrow\rightarrow Z \to Y}$$

- **DE**: Minority patients are more/less likely to die, all other variables kept equal
- **IE**: Minority patients have a higher/lower illness severity
- **SE**: Minority patients are younger/older on average

EP — Non-zero effects may imply inequity

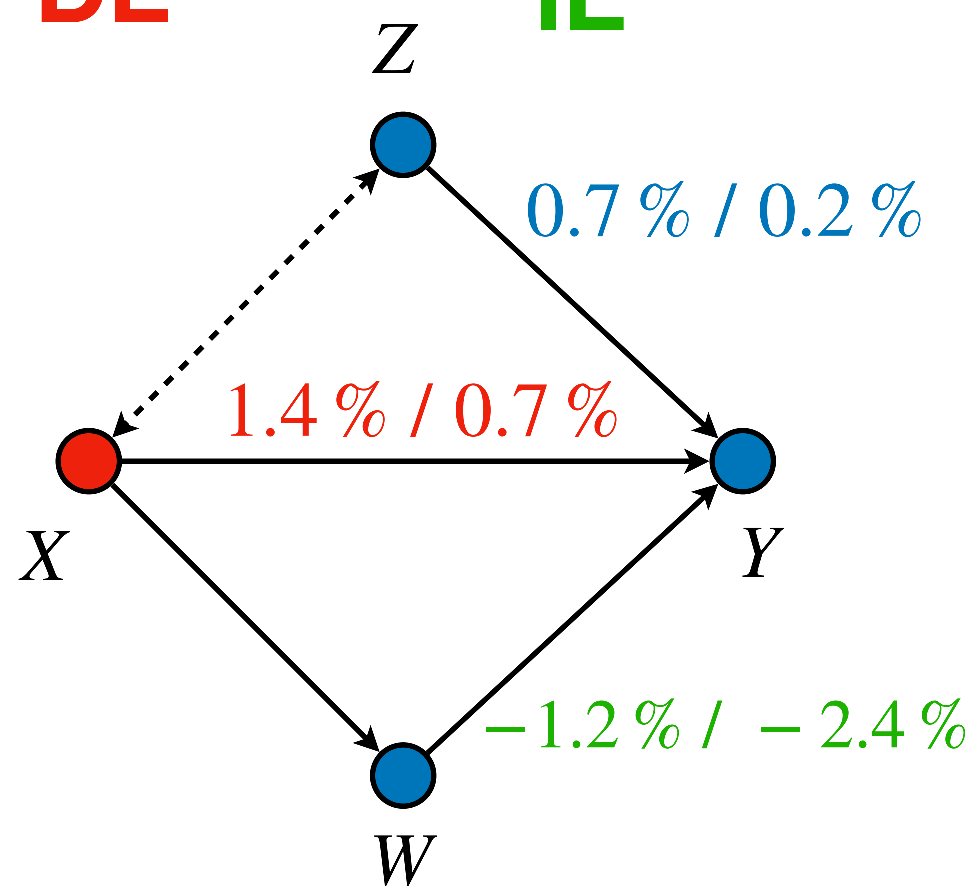
Bias Detection: Hospital Mortality after ICU



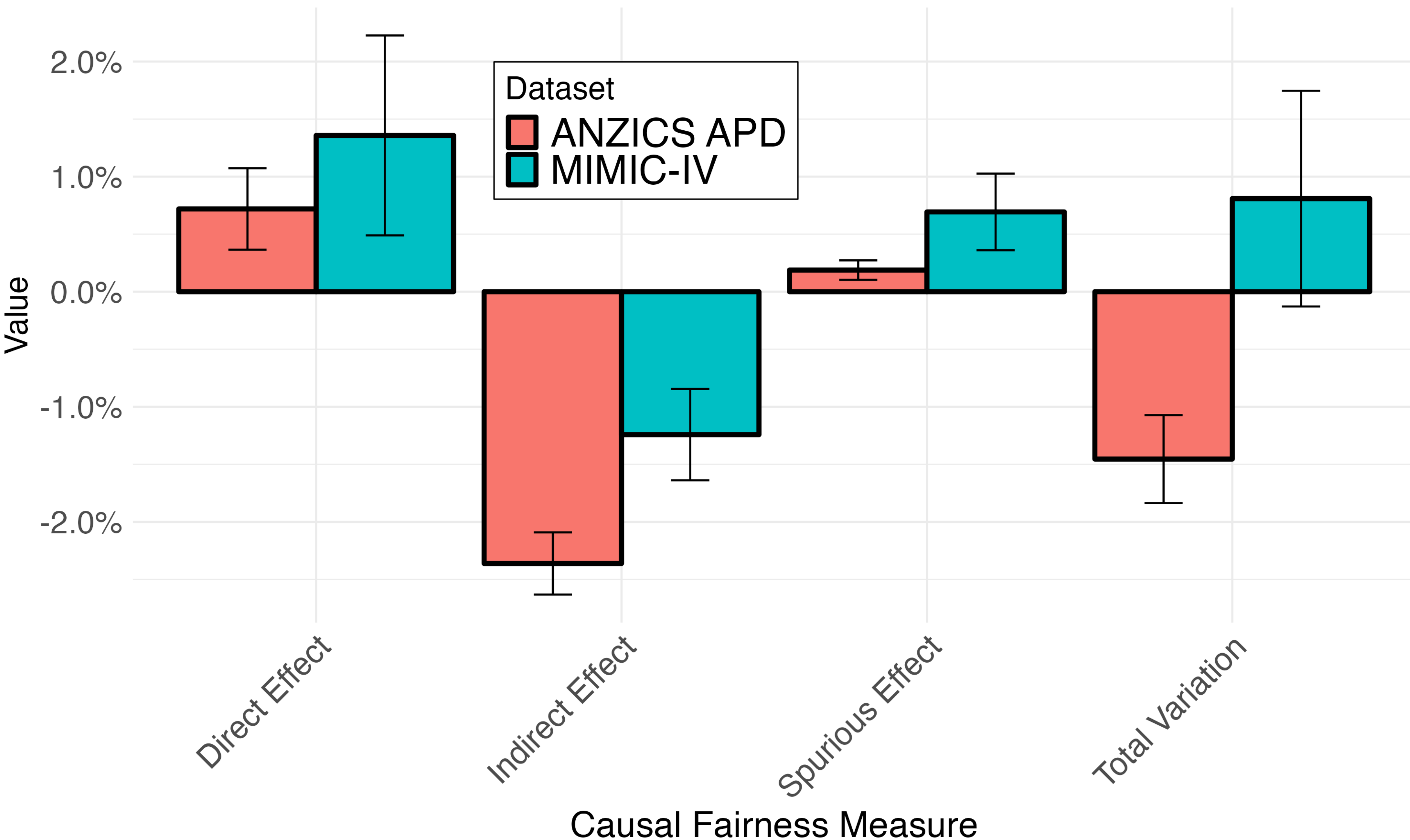
$$\underbrace{0.8\,\%}_{E[Y|x_1]-E[Y|x_0]} = \underbrace{1.4\,\%}_{\textbf{DE}} + \underbrace{(-1.2\%)}_{\textbf{IE}} + \underbrace{0.7\,\%}_{\textbf{SE}}$$

$$\underbrace{-1.5\,\%}_{E[Y|x_1]-E[Y|x_0]} = \underbrace{0.7\,\%}_{\textbf{DE}} + \underbrace{(-2.4\%)}_{\textbf{IE}} + \underbrace{0.2\,\%}_{\textbf{SE}}$$

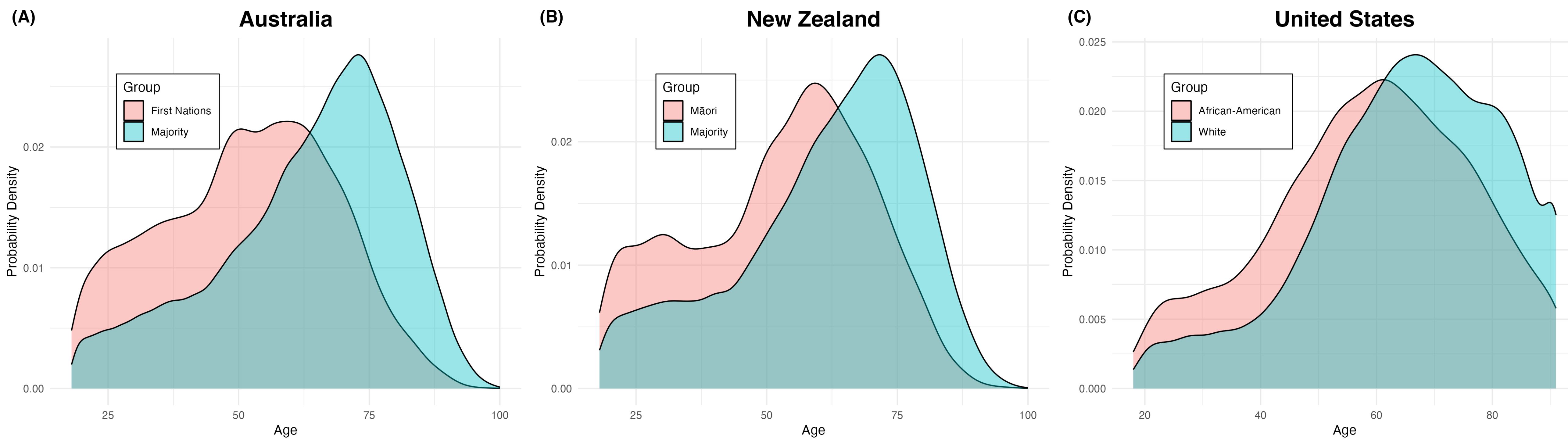
A causal perspective on the problem is clearly more robust

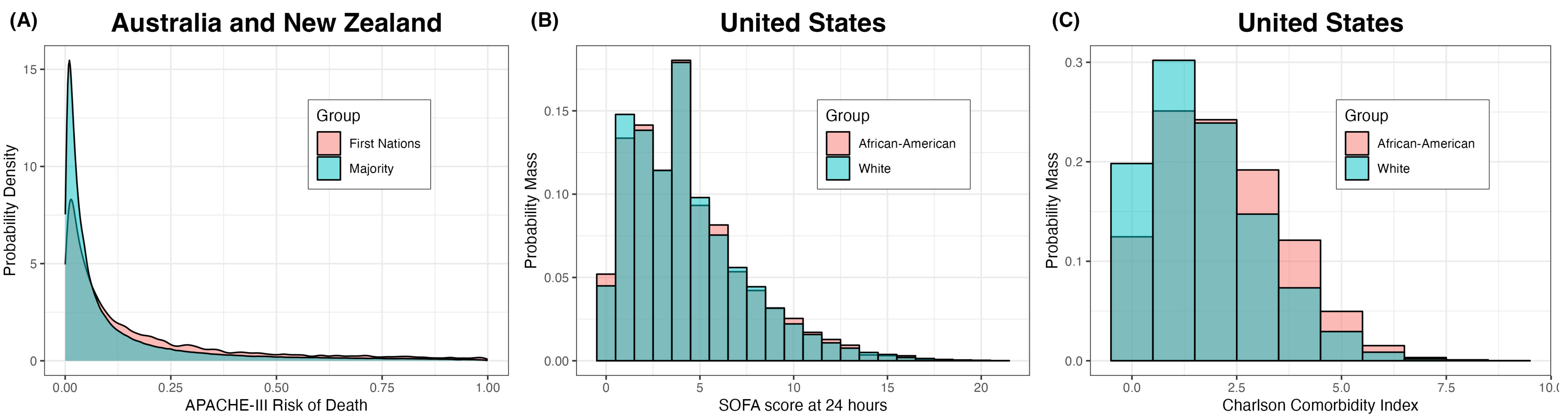


Bias Detection: Hospital Mortality after ICU

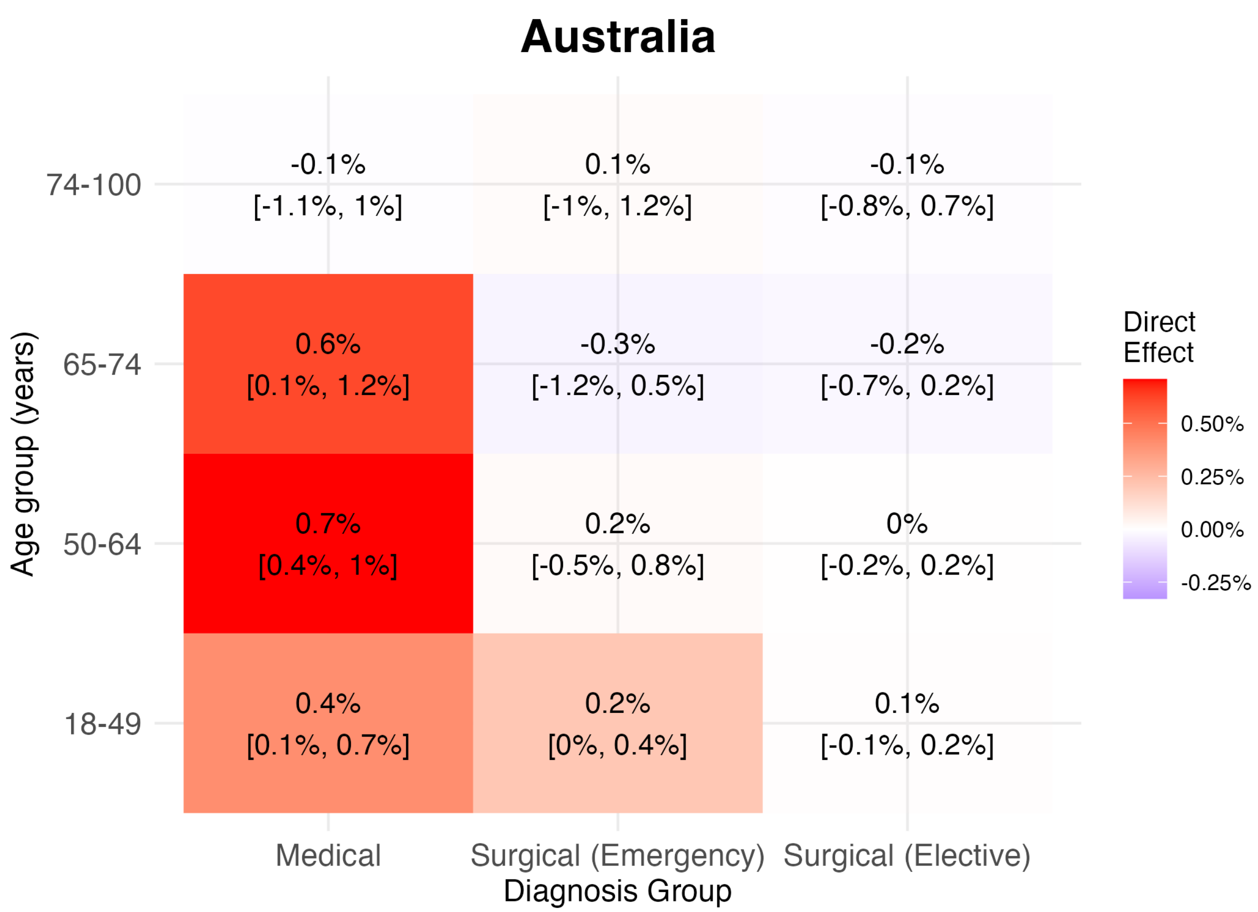


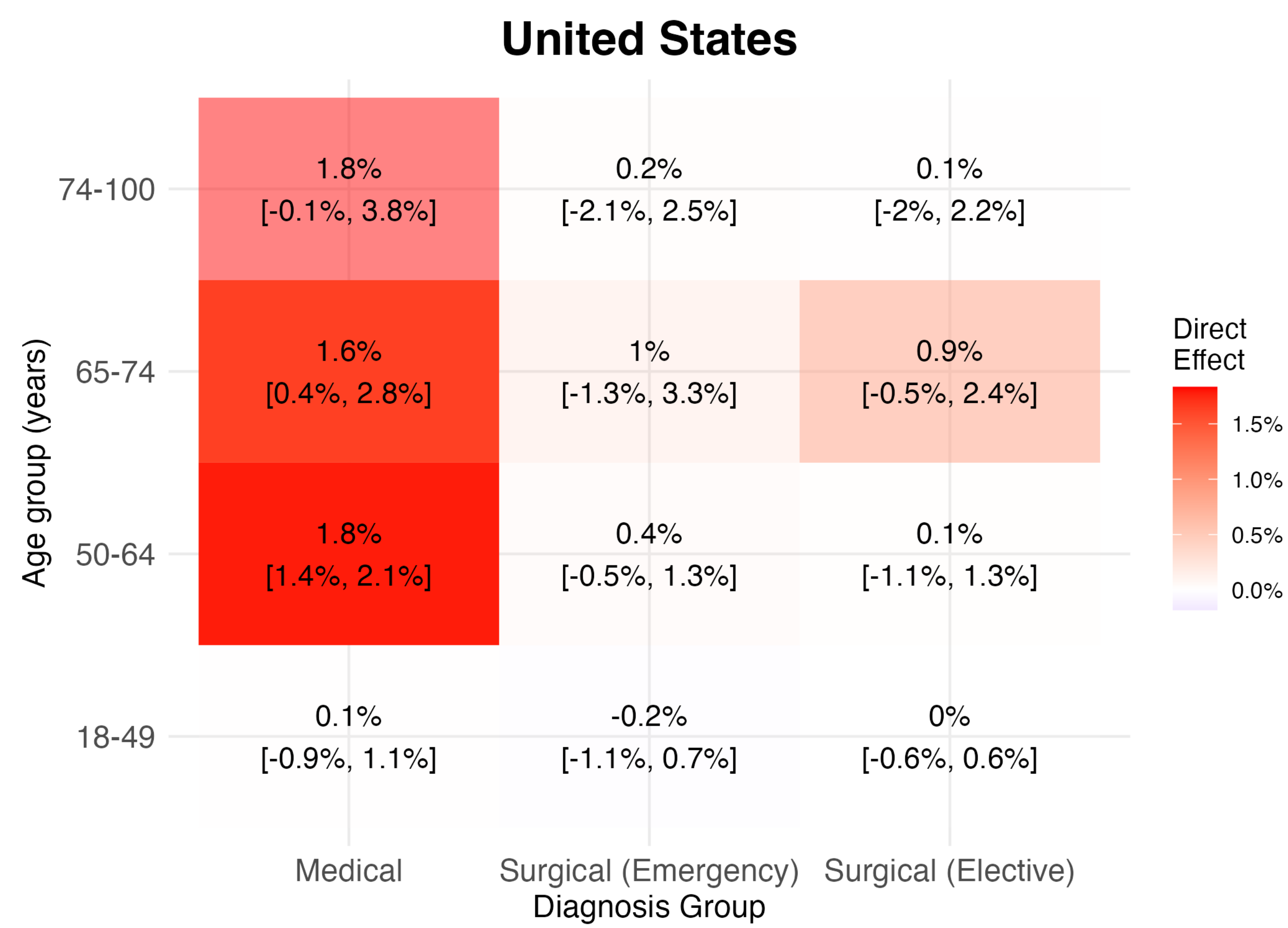
Bias Detection:
How to explain these effects? (IE/SE)





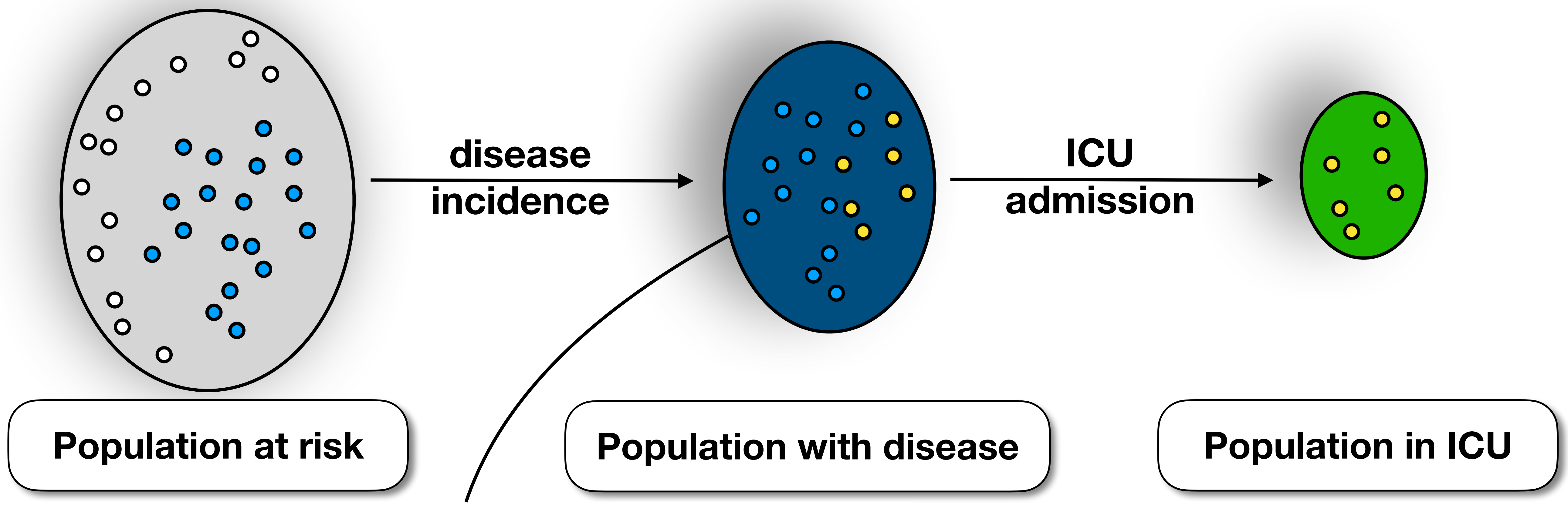
Heterogeneous Direct Effect (Age / Admission Type)

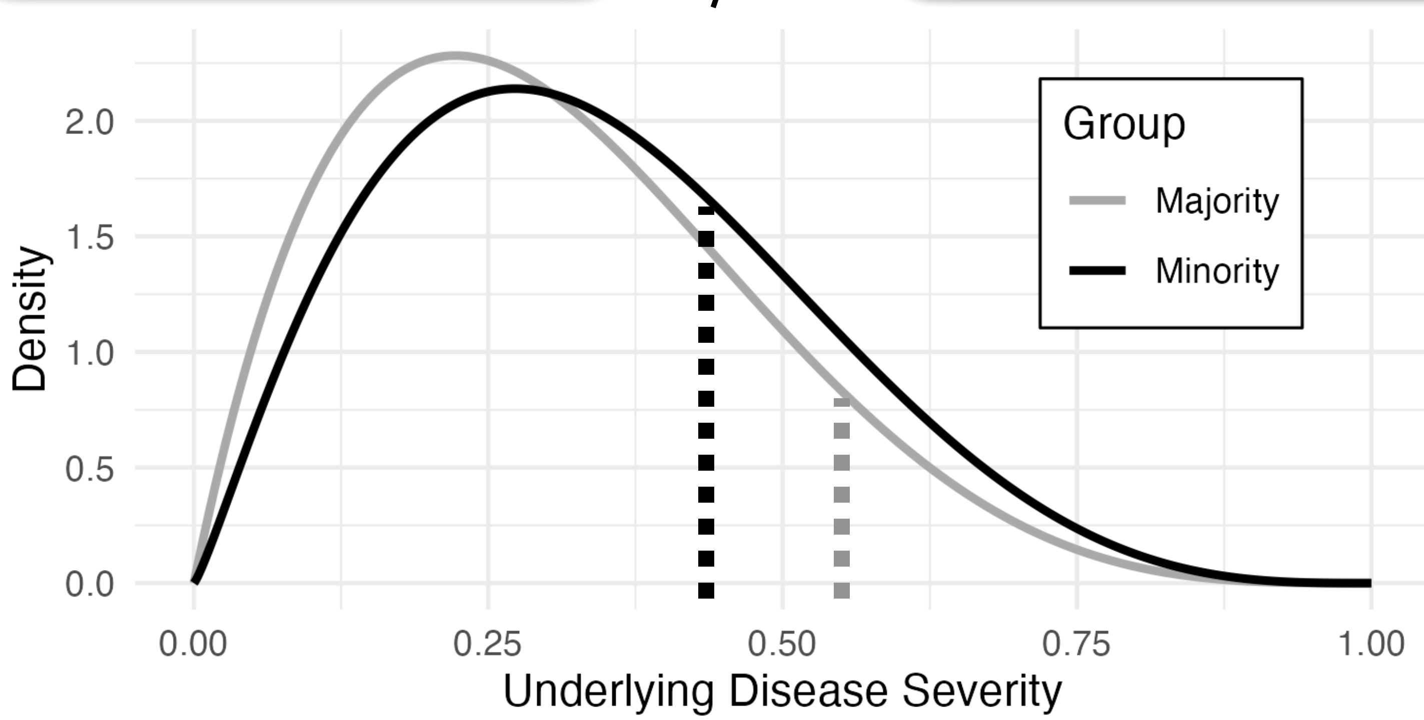




Direct effect driven by *medical* admissions

But why is there a protective effect for medical admissions?





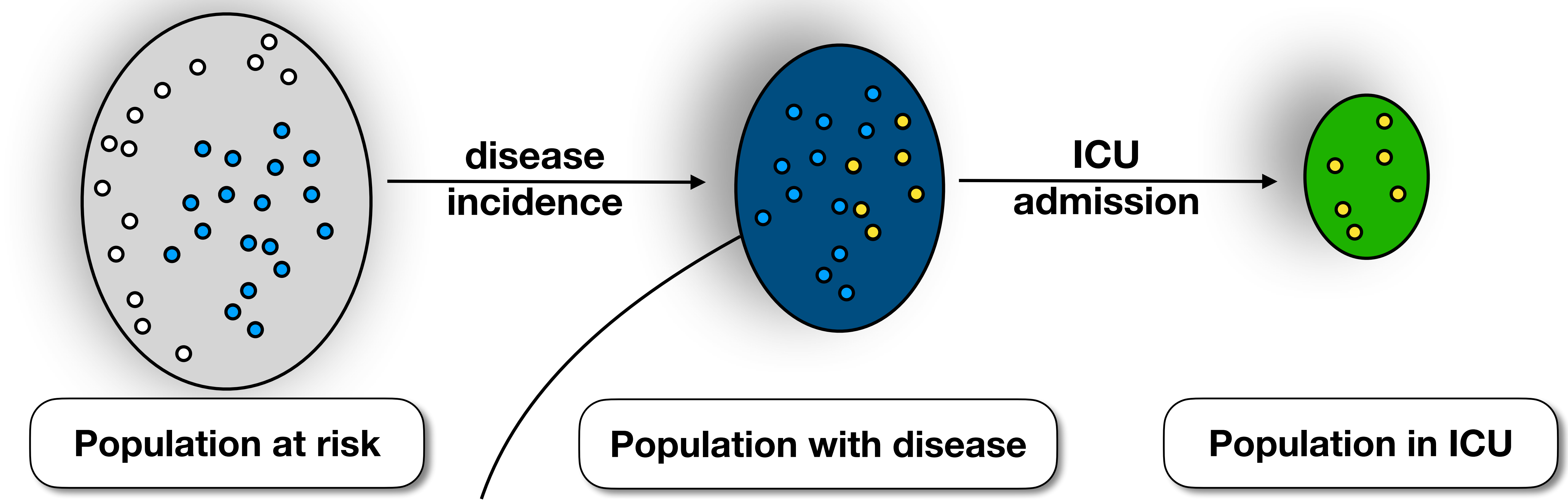
we do not select only the sickest patients

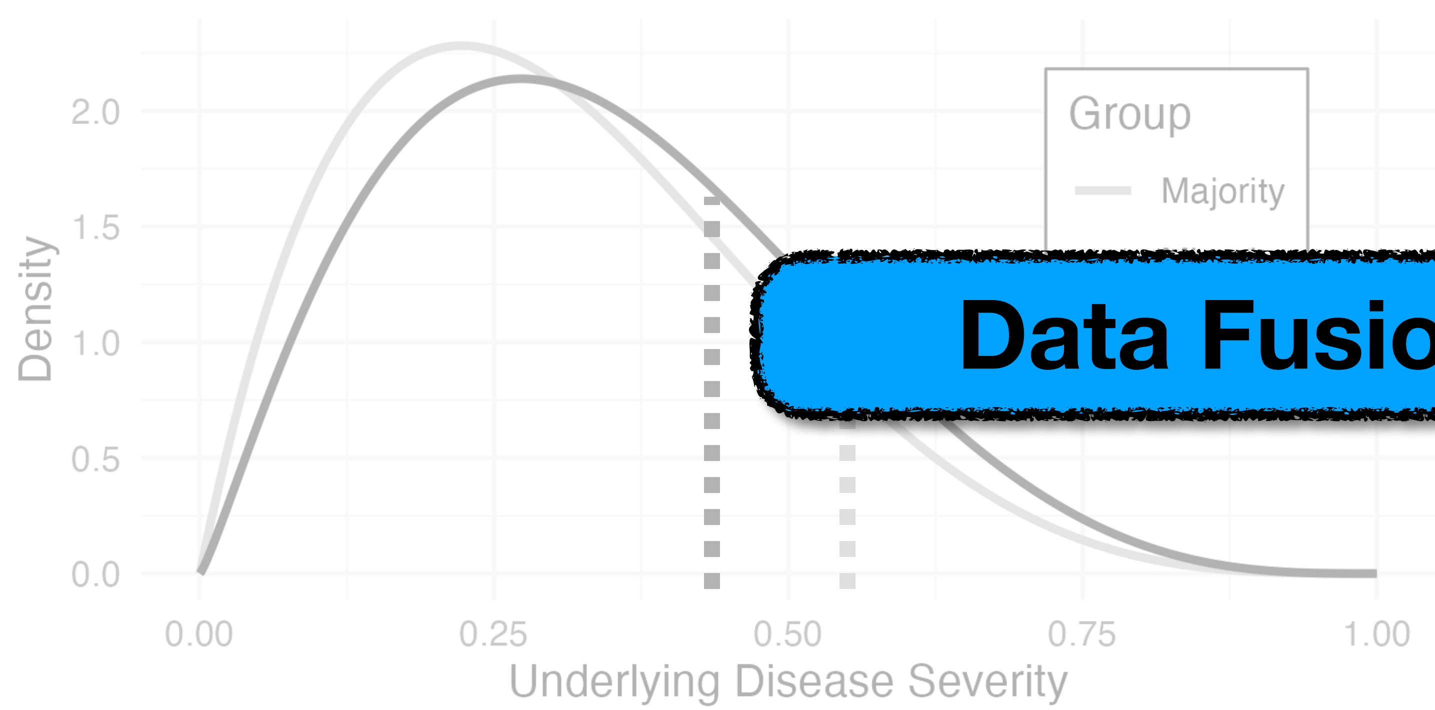
higher prevalence = lower severity

If so, are medical admissions oversampled?

Does it explain the DE?

But why is there a protective effect for medical admissions?





Data Fusion

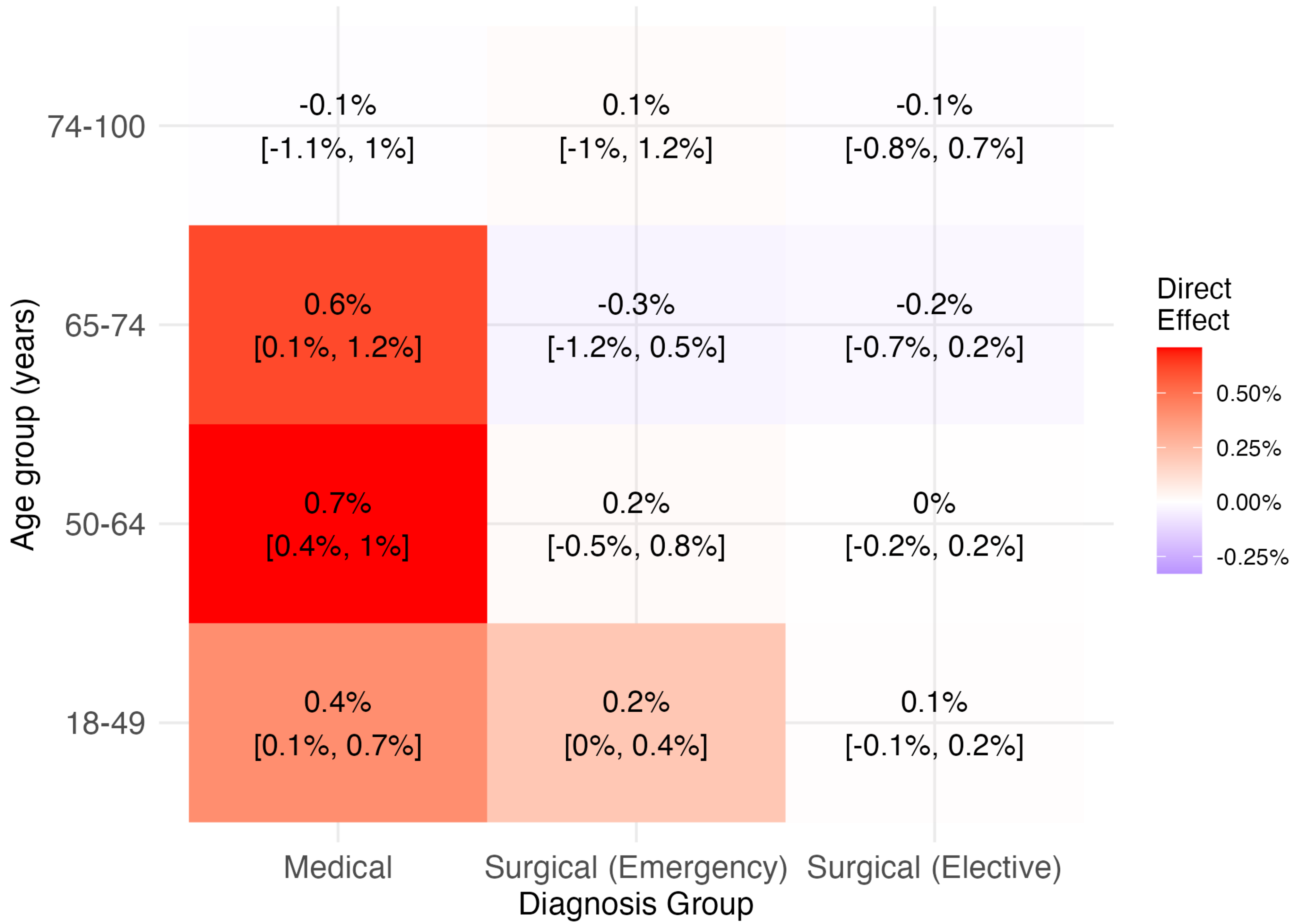
we do not select only the sickest patients

higher prevalence = lower severity

If so, are medical admissions oversampled?

Does it explain the DE?

Baseline Risk & Direct Effects



Indigenous Intensive Care Equity (IICE) Radar